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91B50, 46A, 46N10,

DK-CPNH-EC

In an exchange economy with a finite number of consumers and a continuum of commodities, the paper formulates necessary and sufficient conditions on commodity space that will under common assumptions on consumer behavior, but without restrictions on the consumption set under a monotonicity assumption for a particular consumer, and with a continuity requirement on demand when it is feasible, ensure existence of a competitive equilibrium for an exchange economy. Its results strengthen the notion that “reasonably well behaved” economies have a competitive equilibrium. For the initiated, this paper is written in very readable way, with clear proofs and interpretations. Accessibility for those who tend to avoid topology altogether would have benefitted from a short appendix with main definitions of underlying concepts and notation, but then again this may not be the journal’s policy. More importantly, while the results are well announced in the introduction and seem clear cut, the relation to existing literature remains somewhat vague. Furthermore, it might have been relevant to hint at some specific classes of competitive equilibrium models for which the result might apply: dynamics (infinite horizon dynastic models), under uncertainty (infinite number of financial assets), and product differentiation (infinite number of varieties), and whether the stated assumptions are restrictive in these settings.

I would also mention, though this only is a side remark as it is subject nor aim of the paper, that the result only refers to existence of equilibrium, whereas indeterminacy in my reading is considered the key question (see Kehoe et al., 1989). In a nutshell, the existence proofs in Walrasian tradition show that there are sufficient variables (number of prices minus one) to accommodate an equal number (number of commodities minus one) of constraints. This “minus one” in both variables and equations is obtained via Walras law and homogeneity of degree zero in prices of excess demand. For fixed point theorems this finds expression in the mapping of a compact convex set into itself, which for the Kakutani theorem referred to in the paper has to be upper semicontinuous, convex valued. However, as the number of commodities runs to infinity, this requirement loses meaning since infinity minus a finite constant is neither less nor more than infinity itself. There are in fact quite a few, admittedly less general ways to represent an infinite number of commodities without having to incur these difficulties. For example, one may seek extension of the Lancaster (1966) approach with a fixed characteristic composition per commodity to a form with variable composition. This produces a multidimensional continuum of potential commodities without need for an infinite number of markets, because the number of actual commodities is finite. Such a form definitely achieves less mathematical generality than the one in the paper, but I find it more relevant within economic theory, since it can avoid all indeterminacy and depict economies in which the cost of implementing competitive conditions remains manageable.

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